

# Ch. X: Supplement 1

- Turning  $\hat{H}\psi = E\psi$  into a matrix equation

Let  $\{\varphi_i\}$  be a complete set of functions.

We can expand  $\psi = \sum_i a_i \varphi_i$   
unknown                      unknowns                      known set of functions

$$\hat{H}\psi = E\psi \Rightarrow \sum_i a_i \hat{H}\varphi_i = E \sum_i a_i \varphi_i$$

Multiply by  $\varphi_j^*$  and integrate over all space:

$$\sum_i a_i \int \varphi_j^* \hat{H} \varphi_i d\tau = E \sum_i a_i \int \varphi_j^* \varphi_i d\tau$$

$$\langle j | \hat{H} | i \rangle \equiv H_{ji}$$

$\delta_{ij}$  orthonormal set of  $\{\varphi_i\}$

$$\Rightarrow \boxed{\sum_i H_{ji} a_i = E a_j}$$

$$\begin{pmatrix} H_{11} & H_{12} & H_{13} & \dots \\ H_{21} & H_{22} & H_{23} & \dots \\ H_{31} & H_{32} & H_{33} & \dots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix} \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \end{pmatrix} = E \begin{pmatrix} a_1 \\ a_2 \\ a_3 \\ \vdots \end{pmatrix}$$

Solving for  $E \Rightarrow$  solving for eigenvalues of the matrix  $\hat{H}$  with matrix elements  $H_{ij}$